**Assignment 3**

**Name:** Sai Naga Viswa Chaitanya Basava

**Email:** [sainagaviswachaitanya.basava@utdallas.edu](mailto:sainagaviswachaitanya.basava@utdallas.edu)

**NetID:** SXB220302

**Ques 1. Write a proof by induction to show that power(x, n) correctly returns x^n.**

**power(x, n): // Precondition: x > 1, int n >= 0. Assume no overflows.**

**if n = 0 then**

**return 1**

**else**

**s = power(x\*x, n/2)**

**return n%2==0 ? s : s\*x**

**Base case:** when n = 0, the methods always return 1, which is the correct value for x^0.

**Inductive hypothesis**: Assume power(x, n) returns the correct value (i.e., x^n) for all m, m < n

We need to show that it holds for m = n as well.

There can be 2 cases that are possible:

CASE 1: n is even

From the hypothesis,

And since n is even, power(x, n) will return s, which is equal to

This is the expected output.

CASE 2: n is odd

From the hypothesis, . Since, n is odd, and n/2 is integer division

And since n is even, power(x, n) will return s \* x, which is equal to

This is also the expected output.

Therefore, the inductive hypothesis holds true for all values of n >= 0. Hence power(x, n) will correctly return x^n for all values of n >= 0.

**Ques 2. Prove by induction that the solution to the Merge sort recurrence,**

**given below, is O(nlogn).**

**T(n) <= 2T(n/2) + n, for n>1, T(1) = 1.**

**Write the proof by showing that T(n) <= anlogn + bn, for some constants a and b,**

**and derive the values of a and b from the proof.**

Let for some a, b

**Inductive hypothesis**: Assume this holds for all m, m < n

We need to show it holds for m = n

For all

From base case of when n = 1, we can write [from inductive hypothesis]

From this we can write [since T(1) = 1]

Therefore, the inductive hypothesis holds of true for all values of .

Therefore, the solution to the merge sort’s recurrence is .